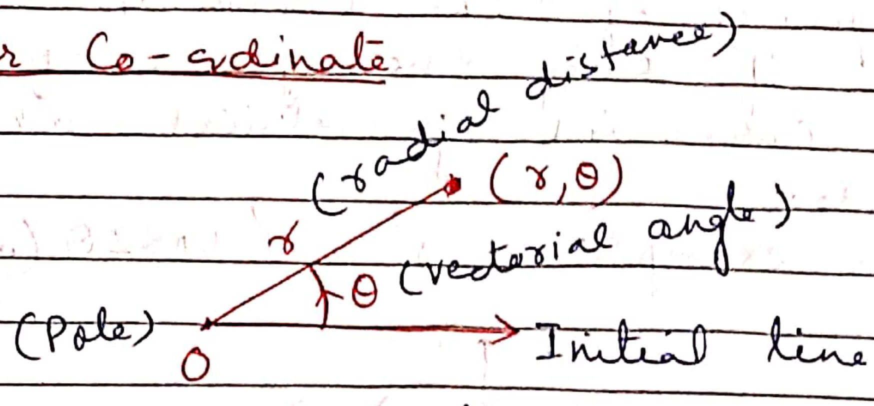
Polar Curves

Polar Co-ordinate



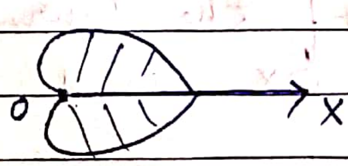
Polar Co-ordinate system is a two-dimensional Co-ordinate system in which each point on a plane is determined by a distance from a reference point and an angle from a reference direction. The reference point is called the pole, If the co-ordinate of a point is (r, θ), Then r is called radius vector and θ is called vectorial angle.

Rules for Tracing the Polar Curves.

(i) Symmetry → (a)

If θ be replaced by $-\theta$ and the equation remains unaltered, the curve is symmetrical about the initial line.

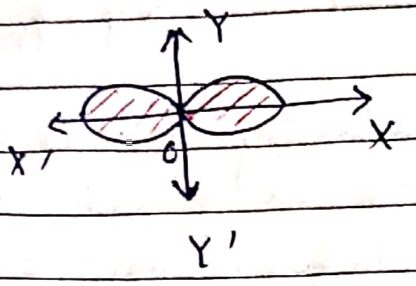
Ex $\quad r = a(1+\cos\theta)$ i.e Cardiode



Here the Cardiode is symmetrical about the initial line.

(b) Replacing r by $-r$, if the equation of the curve is unchanged, i.e. if only even powers of r occur, the curve is symmetrical about the pole.

For example $r^2 = a^2 \cos 2\theta$ (Lemniscate of Bernoulli)



Here the lemniscate of Bernoulli is symmetrical about the pole.

(c) Replace θ by $\pi - \theta$; if the equation of the curve is unchanged, the curve is symmetrical about the line $\theta = \pi/2$ i.e. about the line OY.

(d) The curve is symmetrical about the line $\theta = \frac{\pi}{4}$ if the equation of the curve remains unaltered when θ is changed into $\frac{\pi}{2} - \theta$.

(ii) Intersection of the curve with the initial line OX and with OY.

We find the value of r by putting $\theta = 0$ and $\theta = \pi/2$ in the equation of the curve.

(iii) Whether the curve passes through the pole.

If $r = 0$ for some real value of θ, then curve passes through the pole.
If the curve passes through the pole, the value of θ for which $r = 0$ gives the equation of the tangent at pole.

If $r = 0$ for more than one value of θ, the curve is said to have a loop.

(iv) Region of the curve
To find the region in which no portion of the curve lies, we determine those values of θ for which r is imaginary.

(v) Asymptote if any
If the curve extends up to ∞, then we should also find out the asymptote of the curve.

(vi) Some more Points →
Some specific points on the curve
(a) should be plotted. For this we determine corresponding values of r

for some special values of θ

e.g. $\theta = 0, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}$ ... and plot them.

(b) Sometimes it is convenient to trace out a curve by transforming the curve in the Cartesian form.

Example

1) Trace the curve $y^2(a+x) = x^2(3a-x)$

(i) Since only even powers of y occur, the curve is symmetrical about x-axis.

(ii) The curve passes through origin and tangent at origin is given by $y^2 = 3x^2$ which gives two real and distinct tangents, their equations are given by

$$y = \pm\sqrt{3}\,x.$$

(iii) putting $y=0$, $x^2(3a-x)=0$

$\therefore x=0, 3a$. So, the curve cuts x-axis at $(0,0)$ and $(3a,0)$. Putting $x=0$, we get

$y = 0$, So the curve cuts y axis only at $(0,0)$

(iv) Equating the co-efficient of the highest power of y to zero, the asymptote parallel to y axis is

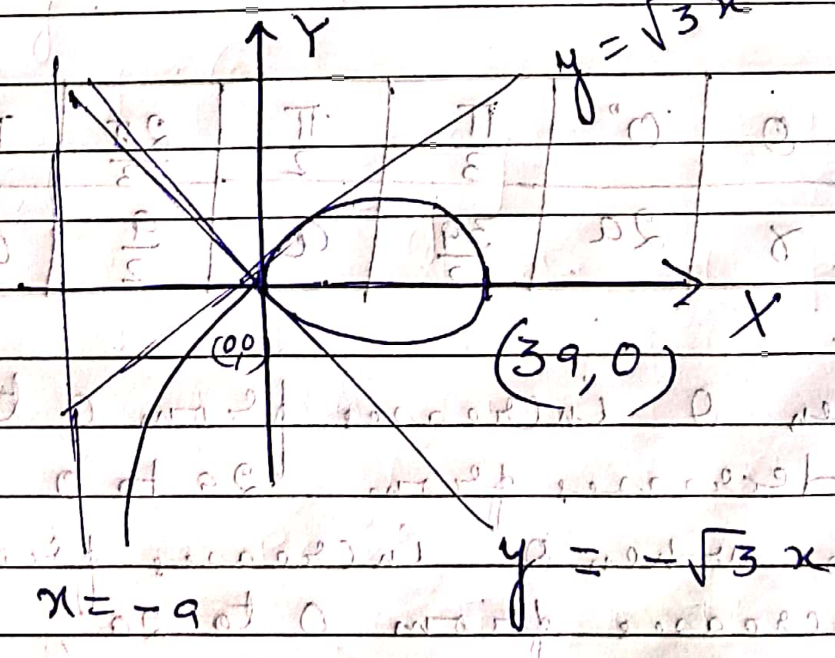
$x = -a$.

(v) Solving for y

$$y = \pm x\sqrt{\frac{3a-x}{a+x}}$$

If $x > 3a$, or $x < -a$, y is imaginary. So, no portion of the curve is in the right of $x = 3a$ & in the left of $x = -a$.

Hence, the curve is



(2) Trace the Cardiode $r = a(1+\cos\theta)$

(i) By changing θ to $-\theta$, we get

$$r = a\left[1+\cos(-\theta)\right]$$
$$= a\left[1+\cos\theta\right]$$

i.e the equation does not change.
So the curve is symmetrical about the initial line.

(ii) when $\theta = \pi$, $r = 0$ hence the curve passes through the pole and the equation of tangent at the pole is $\theta = \pi$, i.e, the initial line.

(iii) Now we plot some points on the curve

θ	$0°$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
r	$2a$	$\frac{3a}{2}$	a	$\frac{a}{2}$	0

when θ increases from 0 to π, r decreases from $2a$ to 0
Also when θ increases from π to 2π, r increases from 0 to $2a$, Since the curve is symmetrical about the initial line.

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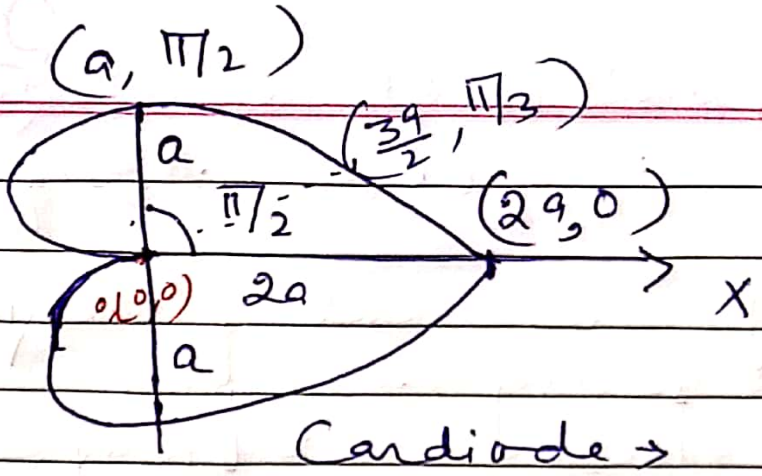

Cardiode → $r = a(1 + \cos\theta)$